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Sound Field Reconstruction at High Frequencies by Using a Piece-wise Interpolation Method

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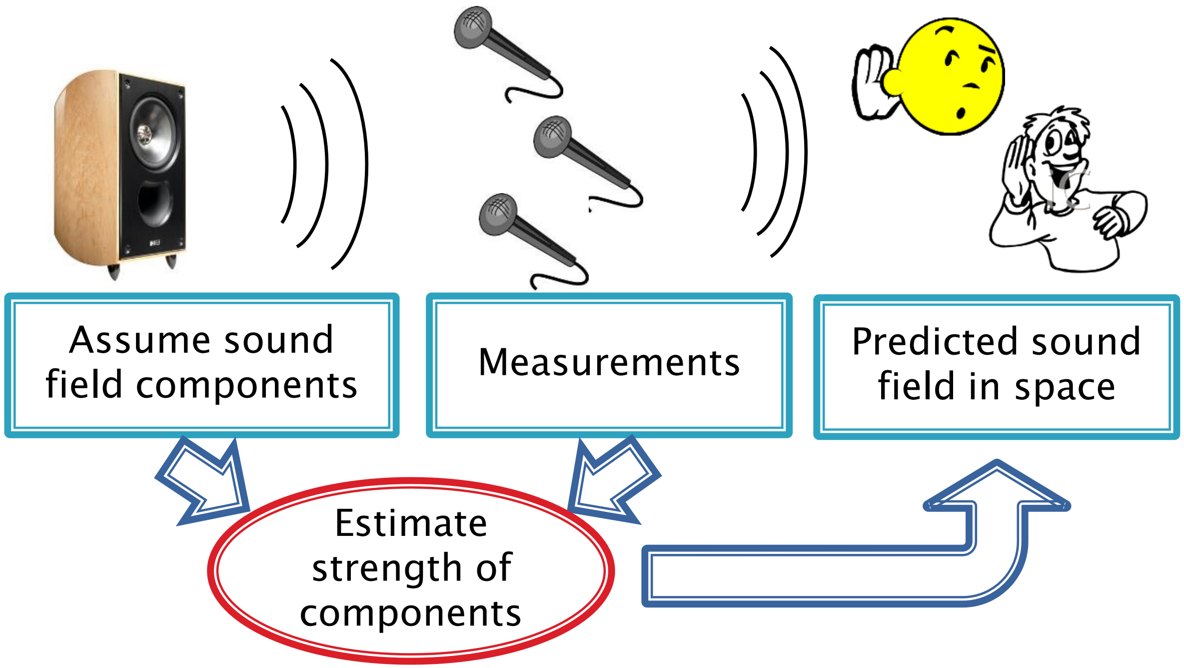
Introduction

- Process of sound field reconstruction
  - The purpose is to predict sound field at any location in space based on measurements.
  - The total sound field is assumed to be composed of several components.



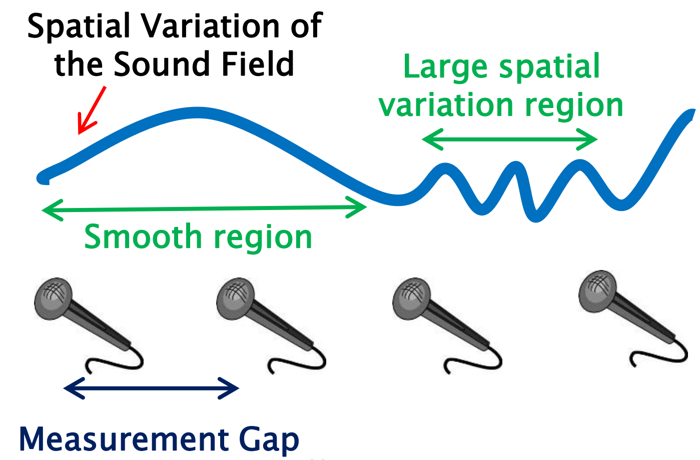
- Challenges at high frequencies
  - The spatial interval in measurements is large compared with wavelength at high frequencies.
  - The spatial variation is usually large at high frequencies.

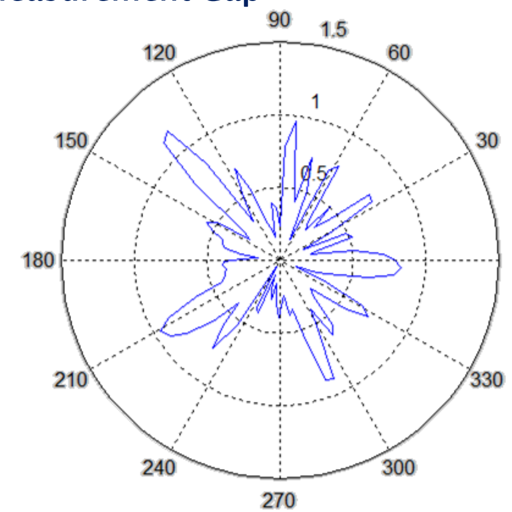
Analyze the Challenge

- The sound field is usually decomposed into components (basis).
- In traditional methods, global basis are used (each component contributes to the whole space)

- Under-sampling in one measurement gap introduces error everywhere in space.

- Use piece-wise basis (contributes to only a certain spatial region) so that we only loose details in some regions.





An Example of Complicated Source Directivity

What piece-wise basis?

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- Approximate representation of high frequency sound field.
  - Spherical wave representation (exact)

$$P=\sum_{n=0}^{\infty}C_{mn}h_n(kr)\sum_{m=-n}^{+n}P_n^m(\cos(\theta))e^{jm\phi}$$

$h_n(\cdot)$ – Spherical Hankel function

$P_n^m(\cdot)$ – Associated Legendre polynomial

  - Spherical Hankel function

$$h_n(z)=(-j)^{n+1}\frac{e^{jz}}{z}\sum_{m=0}^{n}\frac{j^m}{m!(2z)^m}\frac{(n+m)!}{(n-m)!}$$

High frequency:
$kr \gg 1$

$$h_n(kr)\approx\frac{(-j)^{n+1}}{k}\frac{e^{jkr}}{r}$$

Approximated expression:

$$P\approx\frac{e^{jkr}}{r}\sum_{n=0}^{\infty}\sum_{m=-n}^{+n}D_{mn}P_n^m(\cos(\theta))e^{jm\phi}$$

Sound field on the unit sphere determines the whole sound field outside.

Use piece-wise polynomial to represent the sound field on the unit sphere.

Process of the method

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Measurements at arbitrary locations

↓

"Project" measurements onto the unit sphere

↓

Scattered sound field data on unit sphere

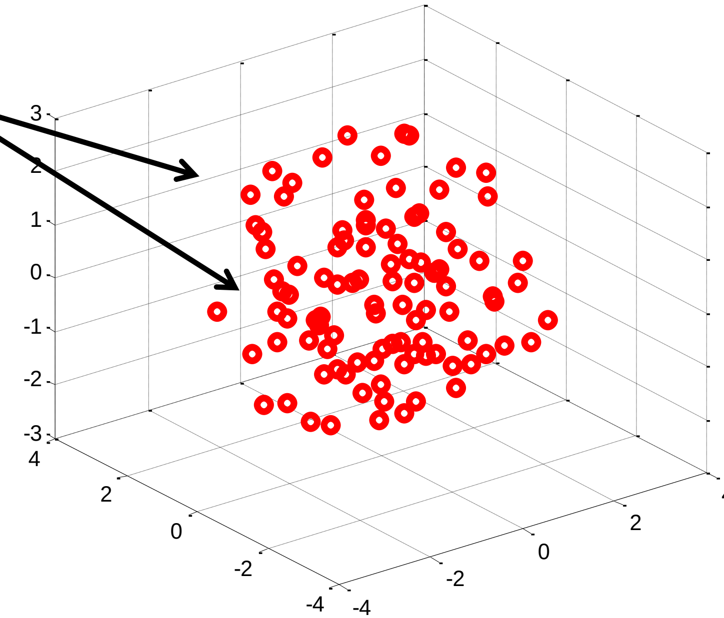
↓

Piece-wise polynomial interpolation on unit sphere

↓

Continuous sound field distribution on unit sphere → "Project" sound field on unit sphere outwards → Predicted sound field in the whole region

- The red circles denote the measurement locations.
- Locations are randomly generated with $1 < R < 3$



- There are **no restrictions** on the locations of the measurements

Process of the method

Measurements at arbitrary locations

⇩

"Project" measurements onto the unit sphere

⇩

Scattered sound field data on unit sphere

⇩

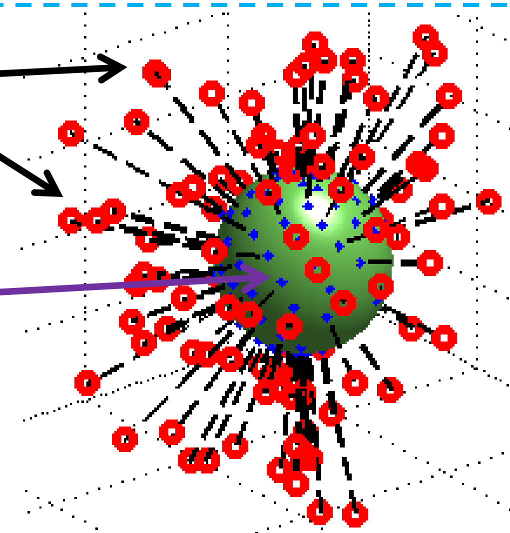
Piece-wise polynomial interpolation on unit sphere

⇩

Continuous sound field distribution on unit sphere ⇨ "Project" sound field on unit sphere outwards ⇨ Predicted sound field in the whole region

Measurement locations (red circles)

Projected points on the unit sphere (blue dots)



Projection means:

- Find the locations on the unit sphere with the same angular coordinates as the measurements
- Calculate the pressure on those locations using:

$$P(1,\theta,\emptyset) = P(r,\theta,\emptyset)\frac{e^{jk(r-1)}}{r}$$

Process of the method

Measurements at arbitrary locations

↓

"Project" measurements onto the unit sphere

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Scattered sound field data on unit sphere

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Piece-wise polynomial interpolation on unit sphere

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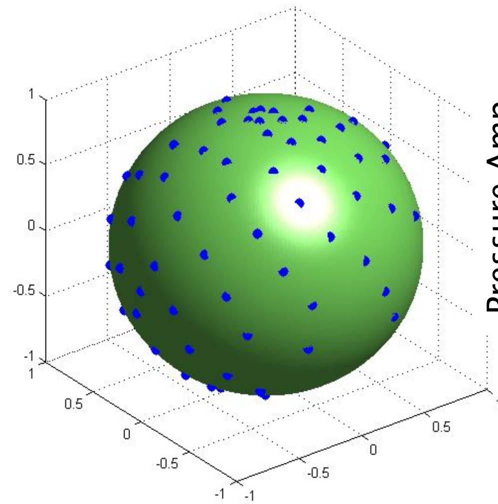
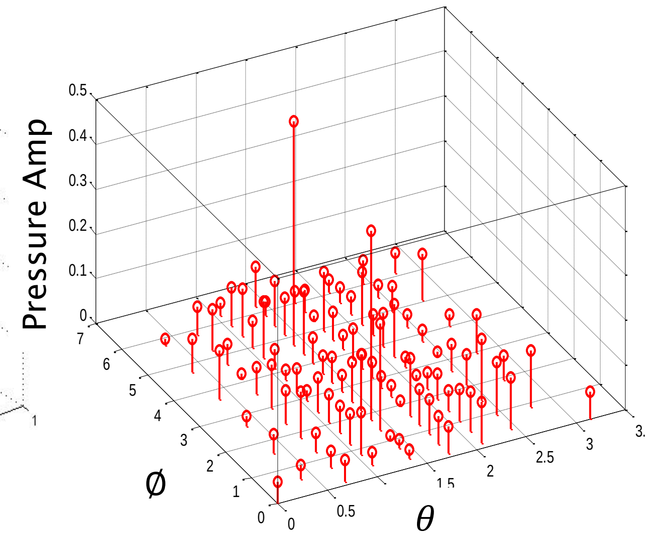
Continuous sound field distribution on unit sphere → "Project" sound field on unit sphere outwards → Predicted sound field in the whole region



Locations of the projected measurements on the unit sphere



Pressure amplitudes of the scattered data (plotted against $\theta, \emptyset$)

Process of the method

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Measurements at arbitrary locations

↓

"Project" measurements onto the unit sphere

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Scattered sound field data on unit sphere

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Piece-wise polynomial interpolation on unit sphere

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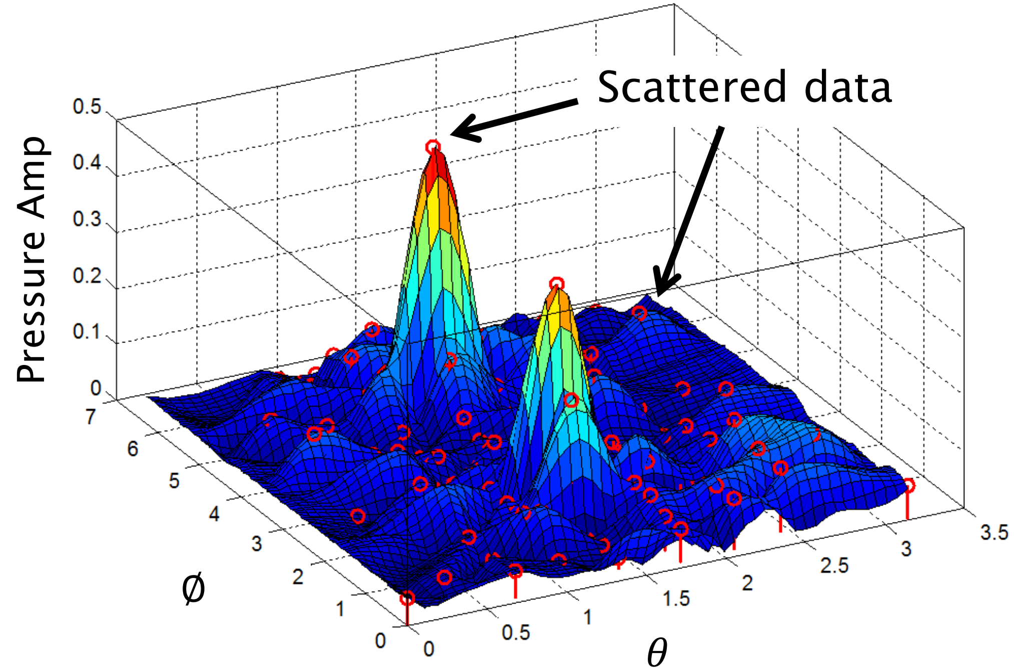
Continuous sound field distribution on unit sphere

→

"Project" sound field on unit sphere outwards

→

Predicted sound field in the whole region



Details of this step will be explained latter in the next section.

Process of the method

Measurements at arbitrary locations

↓

"Project" measurements onto the unit sphere

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Scattered sound field data on unit sphere

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Piece-wise polynomial interpolation on unit sphere

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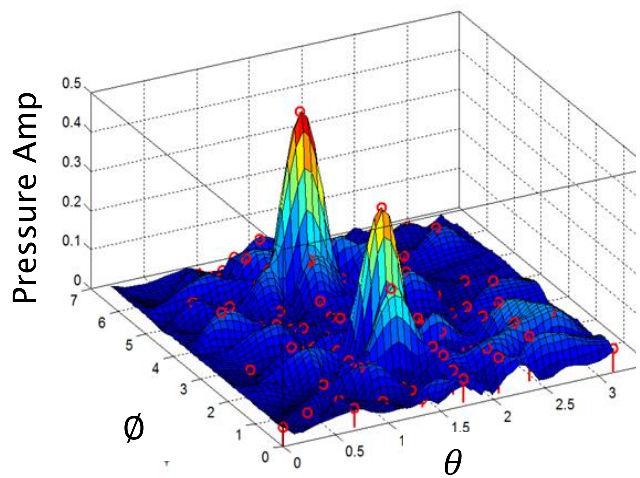
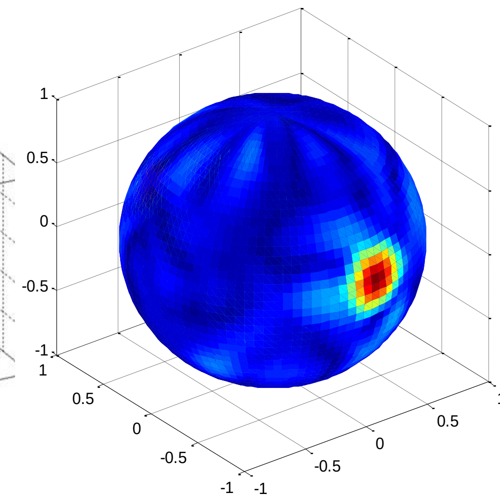
Continuous sound field distribution on unit sphere

→ "Project" sound field on unit sphere outwards

→ Predicted sound field in the whole region



Interpolated sound field distribution on the unit sphere (plot against $\theta, \emptyset$)



Interpolated sound field distribution on the unit sphere (plot on sphere)

Process of the method

Measurements at arbitrary locations

↓

"Project" measurements onto the unit sphere

↓

Scattered sound field data on unit sphere

↓

Piece-wise polynomial interpolation on unit sphere

↓

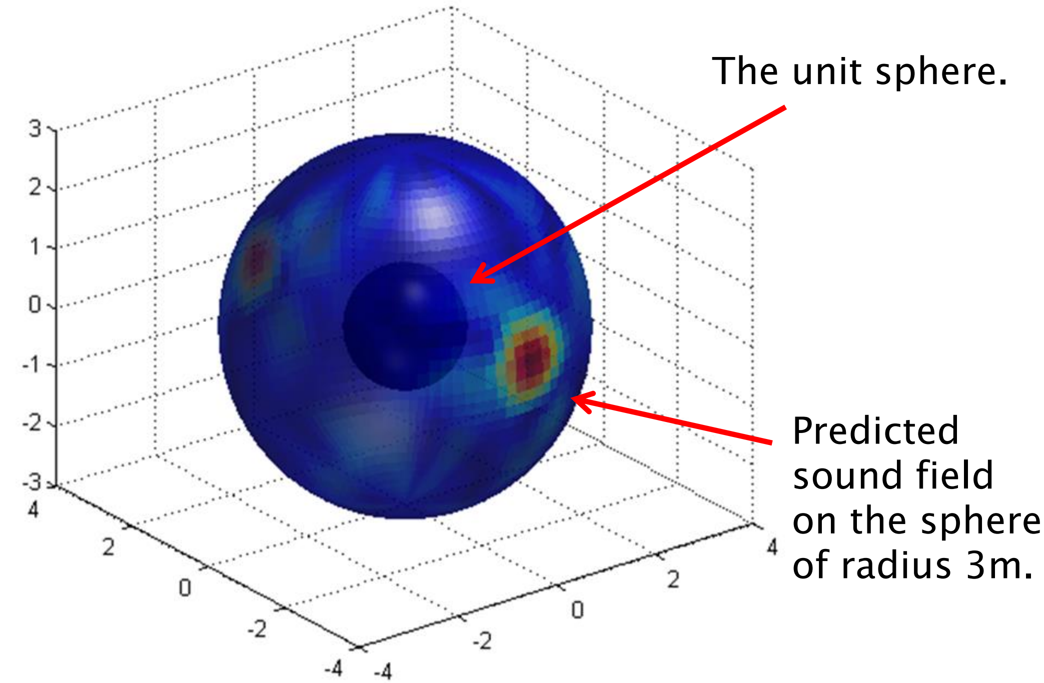
Continuous sound field distribution on unit sphere

→

"Project" sound field on unit sphere outwards

→

Predicted sound field in the whole region



How to interpolate?

- General process of 2D piece-wise polynomial interpolation in Cartesian coordinates

  (1) Connect scattered points into small regions, usually triangles (triangulation).

  (2) Interpolate the data values inside region based on data on the region nodes, usually 2D spline fit.

- Special procedures for interpolation on a sphere

  (1) Triangulation based on Geodesic distance

  R. J. Renka, *Acm. T. Math. Software* **10**(4) (1984).

  (2) Use spherical splines or spherical polynomials

  P. Alfeld, M. Neamtu, and L. L. Schumaker, *J. Comp. Appl. Math.*, **73**(1), (1996).

  P. Alfeld, M. Neamtu, and L. L. Schumaker, *Comput. Aided Geom. D.* **13**(4), (1996).

René Descartes

Simplify the Interpolation

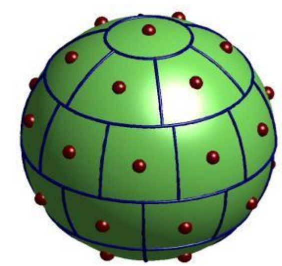
- ❑ Still use regular interpolation tools on spheres

Demonstrate the use of piece-wise basis in sound field prediction at high frequencies.

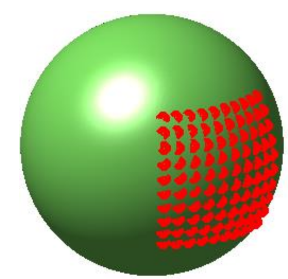
TRICK: Avoid having scattered data concentrated at either of the poles on the sphere.

- **Limit the applications:**

  (1) Predict the sound field in the whole space based on evenly distributed measurements.



  (2) Predict the sound field in a certain region based on measurements in the region. (away from the poles)



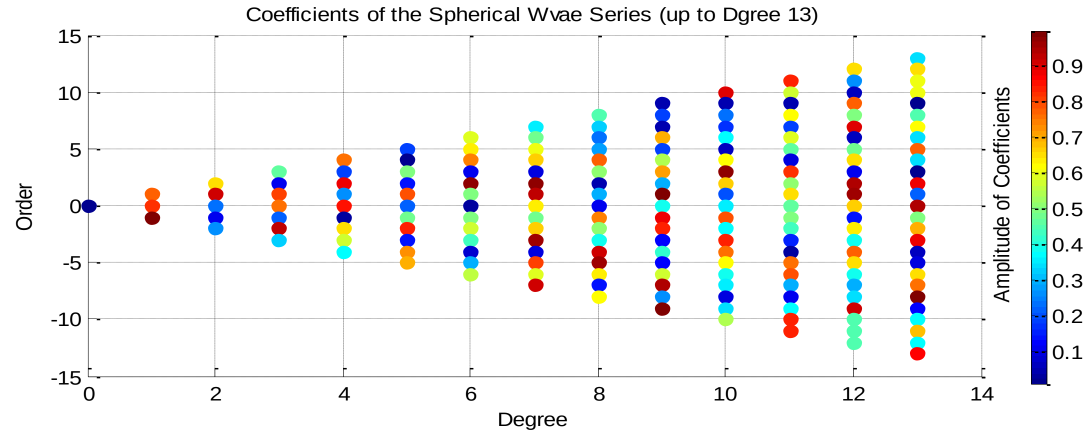
- **Appropriate boundary conditions:**

$$P(\theta,\phi)=P(\theta,\phi+2\pi) \qquad P(\pi,\phi)=const \qquad P(0,\phi)=const$$

Simulation Results

❑ The Generation of Sound Source

- The frequency is 10 kHz.
- Spherical waves are used to generate sound field.
- Source coefficients are generated randomly from uniform distribution.

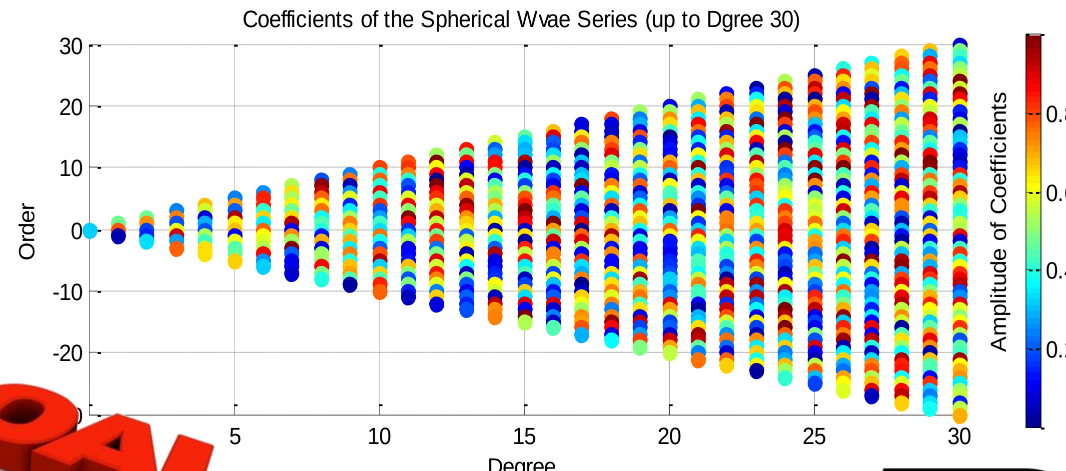


Case (1)

❑ Two Cases:

(1) The number of source terms is less than measurements.

(2) The number of source terms is greater than measurements.



Case (2)

Simulation Results

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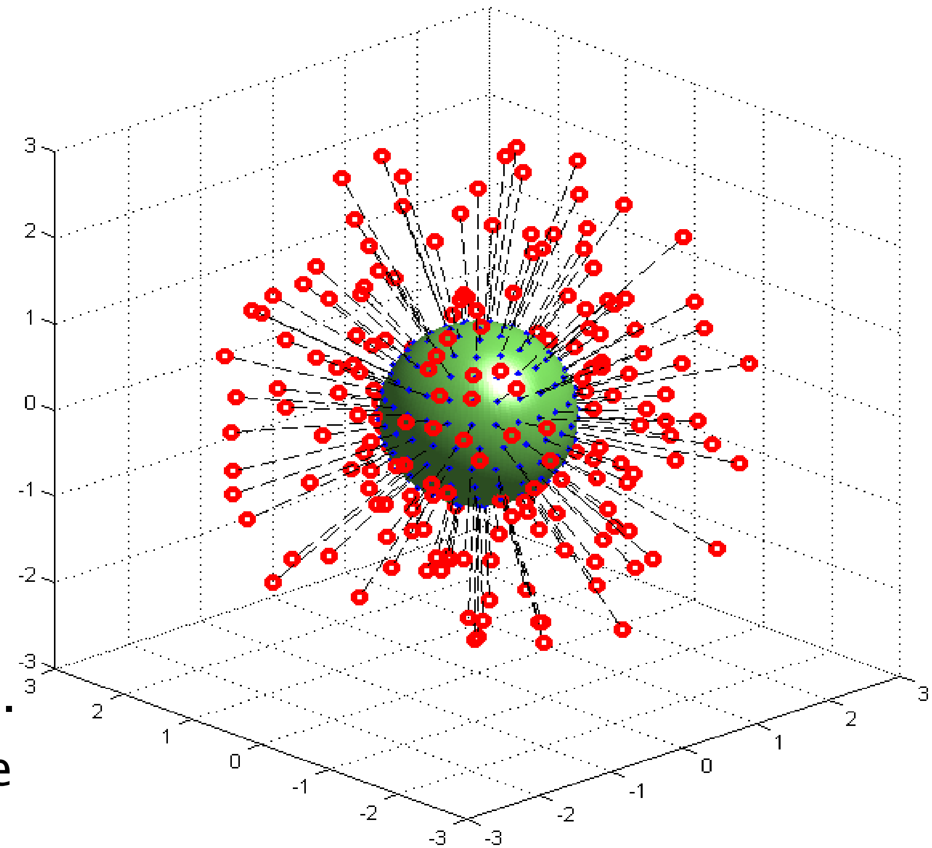
- Reconstruction of sound field in the whole space

  - 200 Measurements.
  - Angular coordinates generated form the Leopardi algorithm. (*P. Leopardi, 2006*)
  - Radius is randomly generated between 1 m and 3 m.

➢ Compare in both cases:

(1) Global basis: spherical waves.

(2) Local basis: piece-wise spline on the unit sphere.



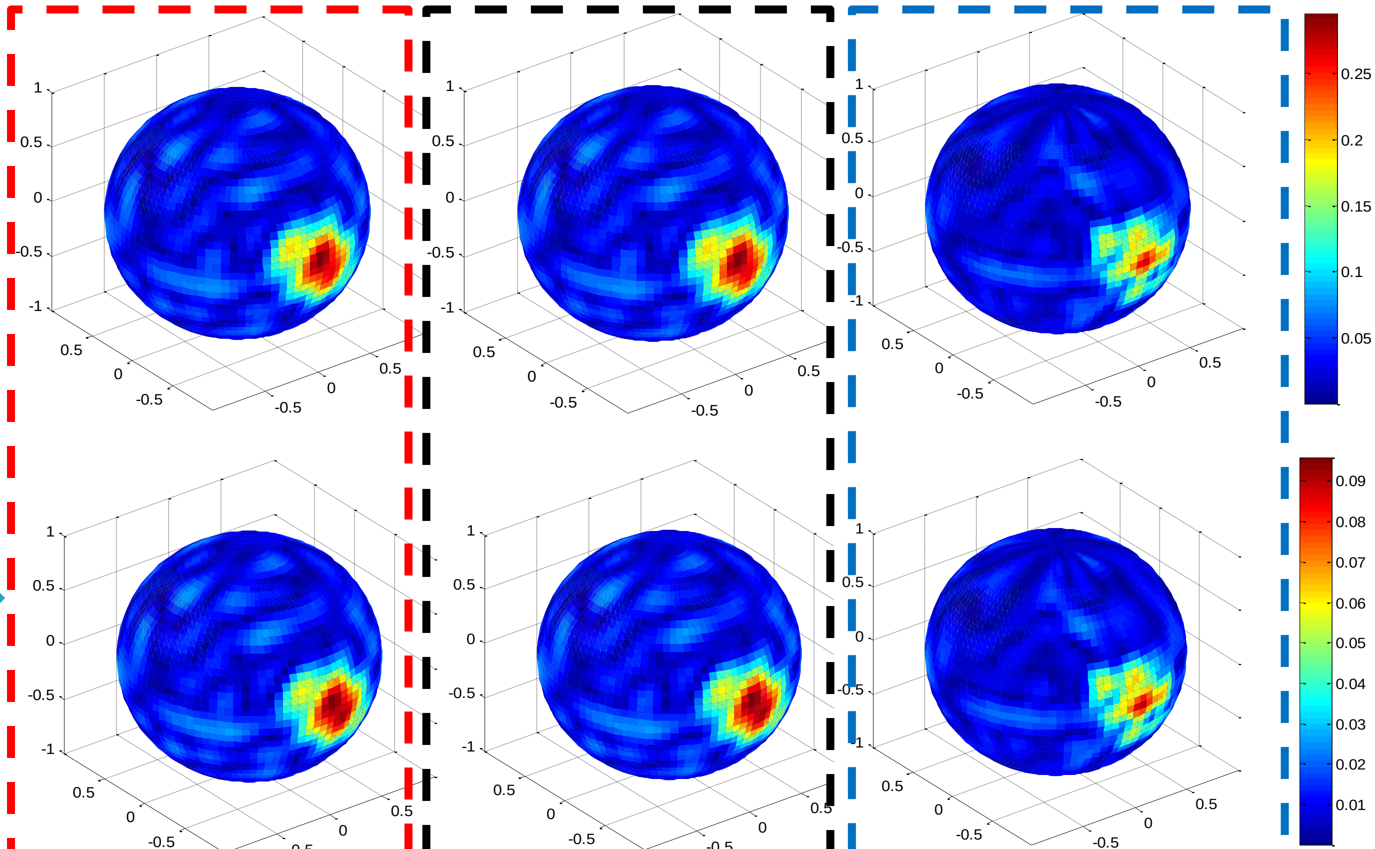
Projection of measurements to the unit sphere

Simulation Results

- Whole space prediction, when the number of source terms is less than the measurements

On the sphere of R = 1 m

On the sphere of R = 3 m



True Sound field

Prediction from spherical wave basis

Prediction from Piece-wise basis

Global basis is better

Simulation Results

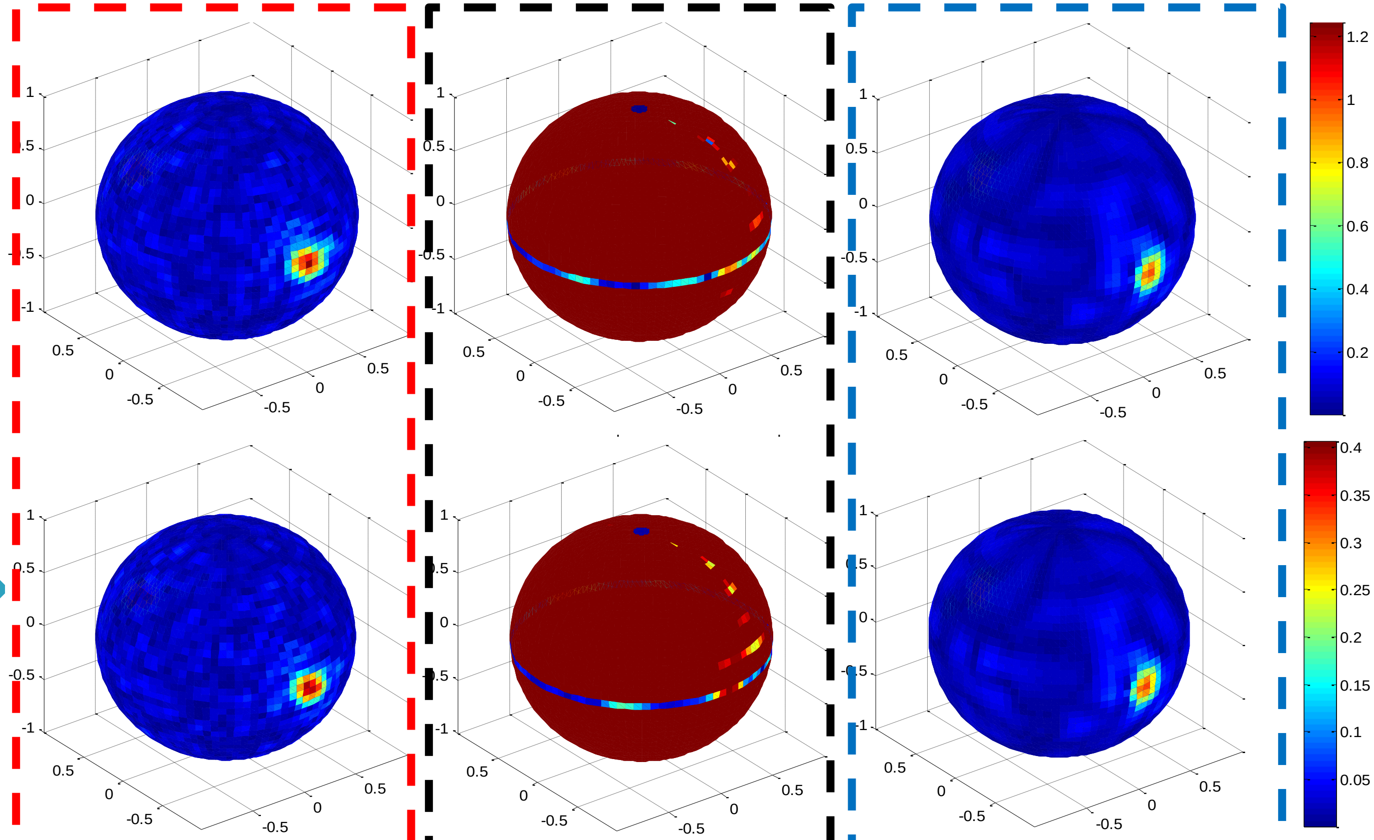
- Whole space prediction, when the number of source terms is much greater than the measurements.

On the sphere of R = 1 m

More Realistic

On the sphere of R = 3 m

Local basis is better



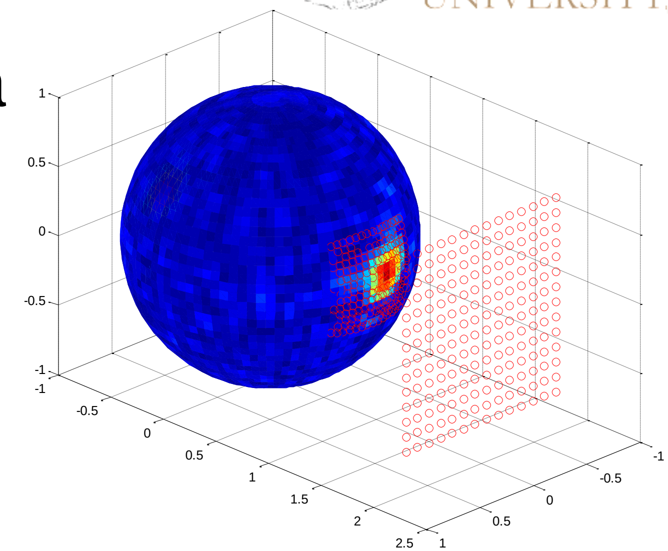
True Sound field

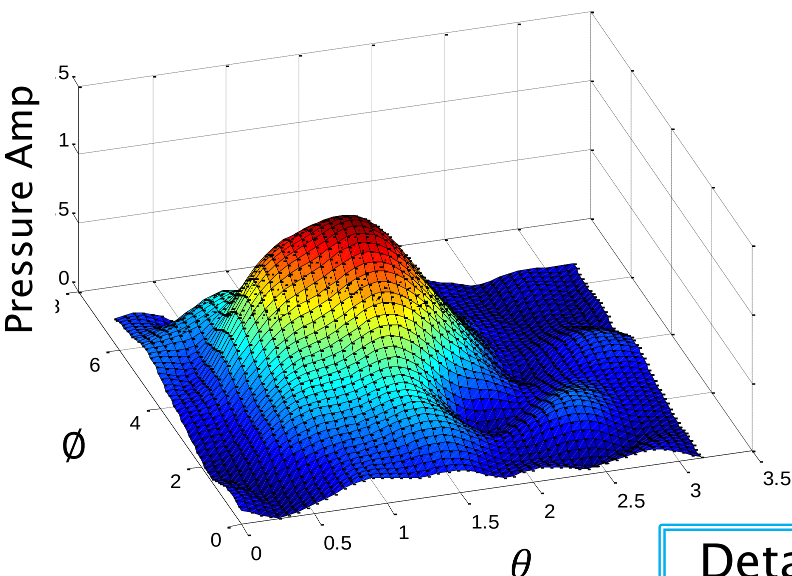
Prediction from spherical wave basis

Prediction from Piece-wise basis

Simulation Results

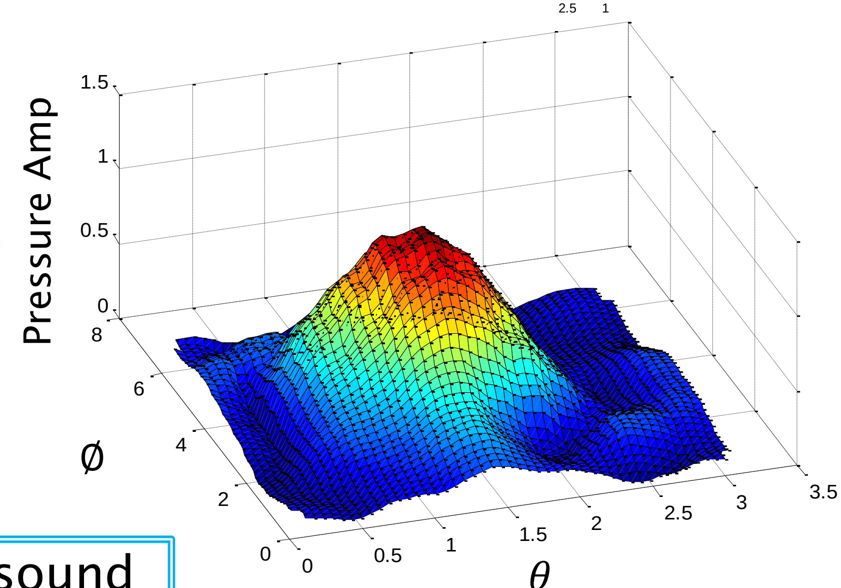
- Reconstruction of sound field in a certain region
  - 200 Measurements are placed in a plane in a certain solid angle region.
  - For the case of the more complicated sound field.
  - Use piece-wise basis.





True sound field

Details of the sound field can be captured



Predicted sound field

Conclusions

- At high frequencies, the sound field can be approximated by the form e^{jkr}/r, except for very small radius.
- The measured sound field can be "projected" on to the unit sphere.
- Sound field on the unit sphere can be reconstructed by piece-wise polynomial interpolation, and then "projected" outward to predict the sound field outside unit the sphere.
- The piece-wise method is shown to be reasonably accurate when the traditional methods fail completely.
- The piece-wise method can also be used to the accurately predict the sound field in certain solid angle region.